However, no contribution from copper was detected in the region behind the shock wave.

Data on test time has been obtained for runs where radiative cooling is unimportant. Values of test time between 2 and 5 μsec were obtained for the conditions covered. These results are approximately a factor of four less than calculations based on the theory of Mirels[12] and continue the trend of increasing divergence between experimental test time and one-dimensional theory for increasing shock velocity shown by Warren and Harris.[9] The test gas slug lengths obtained with the ANAA shock tube are comparable with results obtained with hydrogen helium mixtures in the conical arc driver shock tube,[1,10] although the test times are less due to the higher shock velocities obtained with the ANAA. While the test time has not been definitely measured at the highest speeds produced by the ANAA shock tube, initial analysis by Stickford[13] of the peak radiation and radiative decay rates observed behind the shock wave give good agreement with radiative cooling calculations and indicate that a usable test slug may be produced. Further measurements are needed, however, to show that the 2 μsec of test time needed to simulate the shock layer during Jupiter entry peak heating are obtained in the ANAA shock tube.

Conclusions

An annular arc accelerator shock tube has been built which produces shock velocities and pressures that simulate entry into the atmosphere of Jupiter. Shock velocities up to 47 km/sec have been produced in 1.0 torr of hydrogen which is considerably in excess of the 35 km/sec maximum velocity of conical arc drivers with three times the available energy. The attenuation rate with the ANAA shock tube was found to be comparable with the conical arc driver shock tube, and initial spectroscopic measurements indicate that an impurity free test slug is formed behind the shock wave.

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Adiabatic Ignition of Homogeneous Systems

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Introduction

The thermal model of combustion of premixed gases is frequently employed in the evaluation of the rate of increase of temperature[1,2] and can adequately describe the reaction process in a homogeneous mixture of gases in a constant volume vessel with an initially uniform wall temperature T_o. Moreover if the effect of heat removal is neglected in the onset of explosion, an adiabatic analysis can be assumed.

Assuming an adiabatic process, the first law can be used to derive the following equation for the temperature[1]

$$dT/dt = Z(\bar{c}_v/Q)^{n-1}(T_f - T)^n e^{-E/RT} \tag{1}$$

where T, T_f, and t are dimensional temperature, final adiabatic flame temperature reached by the mixture and dimensional time, respectively. Also Z, Q, and E are the pre-exponential factor, molar efficiency of the fuel (cal per mole) and activation energy of chemical reaction (cal per mole), respectively, and are all characteristics of a particular reaction. Finally R, n, and $\bar{c}_v$ are the universal gas constant, effective order of reaction and mean heat capacity of the mixture at constant volume, respectively.

If Eq. (1) is nondimensionalized in the following manner,

$$\varepsilon = \frac{RT_o}{E}, \quad \theta = \frac{T - T_o}{\varepsilon T_o}, \quad \gamma = \frac{T_o}{T_f - T_o}, \quad \tau = \frac{Z(\bar{c}_v T_o/Q)^{n-1} t}{\varepsilon \gamma^n e^{1/\varepsilon}} \tag{2}$$

we obtain, as elsewhere,[4]

$$d\theta/d\tau = (1 - \varepsilon\gamma\theta)^n e^{(\theta/1+\varepsilon\theta)} \tag{3a}$$

As T is initially equal to T_o, θ must vanish at $\tau = 0$.

$$\theta(0) = 0 \tag{3b}$$

In the following, we shall discuss an approximate solution of Eq. (3) as well as develop an exact analytical solution of the equation. The approximate solution, obtained elsewhere[4] by means of a Zeldovich expansion and a subsequent application of the PLK method, will be shown to be not uniformly valid for all times of interest. The exact analytical solution, which has not been obtained previously, is the major result of this note.

Approximate Solutions of Eq. (3)

Inspection of Eq. (3) shows that the solution exhibits two limiting behaviors:

$$\varepsilon \to 0, \qquad \theta \text{ held fixed}, \qquad \theta \sim -\ln(1-\tau)$$
$$d\theta/d\tau = 0, \qquad \theta \sim 1/\varepsilon\gamma$$

The numerical integration of Eq. (3) has been conducted by Zeldovich and Voevodskii.[3]

Attempts to obtain an approximate solution to Eq. (3) have been based on the smallness of the parameter ε (ε is typically of the order of 10^{-2}). As a result, the well known Zeldovich expansion is frequently used

$$e^{(\theta/1+\varepsilon\theta)} \simeq e^{\theta(1-\varepsilon\theta+\varepsilon^2\theta^2-+\dots)}, \quad \varepsilon\theta \ll 1 \tag{4}$$

Equation (3) is then approximated by

$$d\theta/d\tau = (1-\varepsilon\gamma\theta)^n e^{\theta(1-\varepsilon\theta+\varepsilon^2\theta^2-+\dots)} \tag{5}$$

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Equation (5) was solved approximately by Hermance[4] for two cases (corresponding to retention of the first term and the first three terms in the exponential argument) using the Poincaré Lighthill-Kuo (PLK) method.

Consider the lowest order approximation (in the Zeldovich expansion) to Eq. (5), namely

$$d\theta/d\tau = (1-\varepsilon\gamma\theta)^n e^{\theta} \tag{6}$$

Using PLK method, Hermance[4] has obtained

$$\theta = -ln(1-u) \tag{7}$$

where

$$\tau = u = \sum_{m=1}^{\infty} (\varepsilon\gamma)^m g_m(u) \tag{8}$$

The functions $g_m(u)$ are known analytically and depend upon the chosen value of n. For example, for $n = 1$,

$$g_m(u) = [mg_{m-1} - (-1)^m(1-u)\ln^m(1-u)]; \; g_o = u \tag{9}$$

It is important to recognize that the solution represented by Eq. (7) involves two levels of approximation. The first approximation is the Zeldovich expansion which transforms Eq. (3) to Eq. (6). However, this Zeldovich expansion fails to be valid when θ is of the order of ε^{-1} (e.g. for large times). This failure manifests itself in all solutions obtained by PLK method (regardless of the number of terms retained in the exponential argument) near $\theta = 1/\varepsilon\gamma$. The solution obtained by using Zeldovich expansion is in fact only valid for θ of order unity (for small times) and fails for long times corresponding to the high temperature region. This is clearly demonstrated in Fig. 1 where the exact solutions of Eqs. (3) and (6) (to be obtained later) are compared.

The second approximation involved in obtaining Eq. (7) arises from the use of the PLK method itself. This method, in general, fails to yield correct results in singular perturbation problems where the dependent variable exhibits different orders of magnitude in the range of the independent variable which is of interest. This variation in the order of magnitude of the dependent variable is reflected in the failure of the expansion to be uniformly valid. More specifically, in the present problem the PLK method fails to show the correct asymptotic behavior of the temperature for long times. Hence, although the solution of Eq. (6) obtained by means of the PLK method agrees well with the exact solution for nondimensional times of order unity or less, it fails to show the long time limiting behavior of the solution [as $\tau \to \infty$, $d\theta/d\tau \to 0$ and $\theta \to (\varepsilon\gamma)^{-1}$]. The solution obtained by PLK method is compared with the exact solution of Eq. (6) in Fig. 2. This failure of the PLK method can be confirmed in a different manner. If one rewrites Eq. (3), so that $\theta = 1/\varepsilon\gamma$ corresponds to $\hat{\theta} = 0$, that is

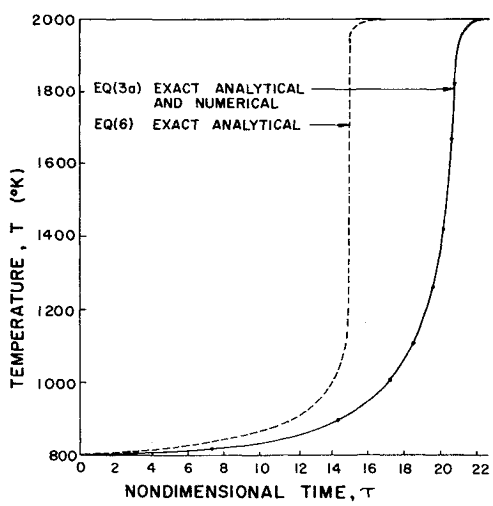

Fig. 1 Comparison of temperature variation with time from exact solution of Eq. (6) with exact and approximate PLK solutions of Eq. (3).

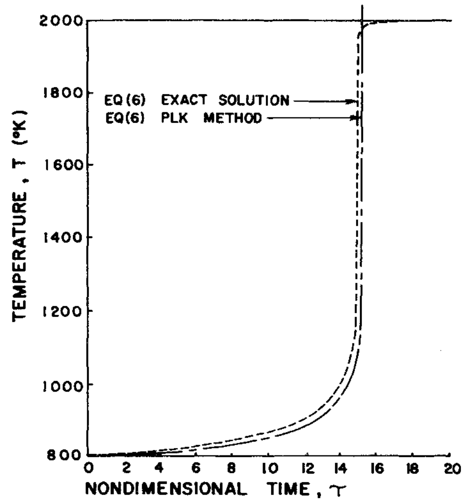

Fig. 2 Comparison of temperature variation with time from exact and PLK solutions of Eq. (6).

$$\theta = \hat{\theta} + 1/\varepsilon\gamma \tag{10}$$

then $\theta(\infty) = 1/\varepsilon\gamma$ implies $\hat{\theta}(\infty) = 0$. Using the Zeldovich expansion in a manner similar to that employed in deriving Eq. (6), we obtain

$$\rho(\varepsilon)[d\hat{\theta}/d\tau] = -(\hat{\theta})^n e^{(\gamma/\gamma+1)\hat{\theta}}, \quad \hat{\theta}(0) = -(\varepsilon\gamma)^{-1} \tag{11}$$

where

$$\rho(\varepsilon) = (\varepsilon\gamma)^{-n} e^{-\gamma/\varepsilon(\gamma+1)} \tag{12}$$

tends to zero as ε tends to zero (for any n).

Equation (11) can be recognized as belonging to the class of singular perturbation problems where a small parameter [here $\rho(\varepsilon)$] multiplies the highest derivative. The PLK method has been shown to fail to yield a uniformly valid expansion everywhere.[5,6] Hence we expect the PLK solution to be invalid when $\hat{\theta}$ is of order unity (i.e., θ of order ε^{-1}).

Exact Solutions of Eq. (3)

We proceed to obtain an exact analytical solution for the approximate [Eq. (6)] as well as the exact equation [Eq. (3)]. To solve Eq. (6), we introduce a new nondimensional time scale, η, as well as a new nondimensional temperature, θ_T as follows

$$\eta = (1-\tau)/f(\varepsilon), \qquad \theta_T = -\theta + 1/\varepsilon\gamma \tag{13}$$

substitution of above in Eq. (6) can be shown to give

$$d\theta_T/d\eta = \theta_T{}^n e^{-\theta_T} \tag{14}$$

where $f(\varepsilon)$ is

$$f(\varepsilon) = (\varepsilon\gamma)^{-n} e^{-1/\varepsilon\gamma}$$

and the initial condition becomes

$$\theta_T[1/f(\varepsilon)] = 1/\varepsilon\gamma \tag{15}$$

Using subsequent integration by parts, Eq. (14) can be integrated, for any integer n, to give

$$\eta = -\sum_{k=1}^{n-1} \frac{\theta_T{}^{k-n} e^{\theta_T}(n-k-1)!}{(n-1)!} + \frac{1}{n!} Ei(\theta_T) + c$$

where $Ei(\theta_T)$ is the exponential integral function defined as

$$Ei(\theta_T) = \int_{-\infty}^{\theta_T} e^t/t \, dt$$

and c is the constant of integration. Using the initial condition Eq. (15) to evaluate c, we obtain the exact analytical solution of Eq. (6), for any integer n as (in terms of τ and θ)

$$\tau = \frac{e^{-(1/\varepsilon\gamma)}}{(\varepsilon\gamma)^n n!}\left[Ei\left(\frac{1}{\varepsilon\gamma}\right) - Ei\left(-\theta + \frac{1}{\varepsilon\gamma}\right)\right] + \sum_{k=1}^{n-1} \frac{(n-k-1)!}{(n-1)!(\varepsilon\gamma)^k} \times [(1-\varepsilon\gamma\theta)^{k-n} e^{-\theta} - 1] \tag{16}$$

For the special case $n = 1$, we have

$$\tau = \frac{e^{-1/\varepsilon\gamma}}{\varepsilon\gamma}\left[Ei\left(\frac{1}{\varepsilon\gamma}\right) - Ei\left(-\theta + \frac{1}{\varepsilon\gamma}\right)\right] \tag{17}$$

Using the results in Ref. 7 for the asymptotic behavior of exponential integral function for small and large values of its argument, we obtain

$$\tau \simeq 1 - e^{-\theta} \quad \text{for} \quad 1/\varepsilon\gamma \gg 1,\ \theta = \vartheta(1)$$

and

$$\tau \to \infty \quad \text{for} \quad -\theta + 1/\varepsilon\gamma \to 0 \tag{18}$$

The latter limit is not obtained by the PLK method. The result given by Eq. (17) is shown in Figs. 1 and 2, and can be compared there with the exact solution of Eq. (3) and the PLK solution of Eq. (6). As a final comparison of the exact solution of Eq. (6) with the approximate solution obtained by PLK method, we note that when

$$\tau = 1 + \varepsilon\gamma + 2(\varepsilon\gamma)^2 + 6(\varepsilon\gamma)^3 + \ldots \tag{19}$$

Eq. (17) implies (for $\varepsilon\gamma \to 0$),

$$Ei\left(-\theta + \frac{1}{\varepsilon\gamma}\right) = 0 \tag{20}$$

or since $Ei(x) = 0$ when $x \simeq 0.37$, we have

$$\theta = 1/\varepsilon\gamma - 0.37 \tag{21}$$

a finite value which the one-term PLK solution fails to yield. While an improvement in the agreement of the PLK and exact solutions can be obtained by including three terms in the Zeldovich expansion,[4] this improved PLK solution still does not yield a finite value of θ for all time τ.

We now wish to develop the exact analytical solution to the exact equation [Eq. (3)] for any integer value of n. We introduce a new nondimensional temperature θ_e defined by the following

$$\theta_e = 1/\varepsilon(1 + \varepsilon\theta) \tag{22}$$

which, upon substitution in Eq. (3), can be shown to give the following differential equation

$$d\tau = -\varepsilon^{n-2}\left(\frac{\alpha}{\gamma}\right)^n e^{-1/\varepsilon}\frac{e^{\theta_e}}{\theta_e^{\,2-n}(\theta_e - \alpha)^n}\,d\theta_e \tag{23}$$

where

$$\alpha = \gamma/[\varepsilon(1+\gamma)] \tag{24}$$

For $n = 1$, Eq. (8) becomes, upon integration

$$\tau + c = 1/(\varepsilon\gamma\, e^{1/\varepsilon})[Ei(\theta_e) - e^{\alpha}Ei(\theta_e - \alpha)] \tag{25}$$

where c is the constant of integration and is determined from the initial condition

$$\theta_e(0) = 1/\varepsilon. \tag{26}$$

The solution for $n = 1$ written in terms $\hat{t} = \varepsilon\gamma\, e^{1/\varepsilon}\,\tau$ becomes

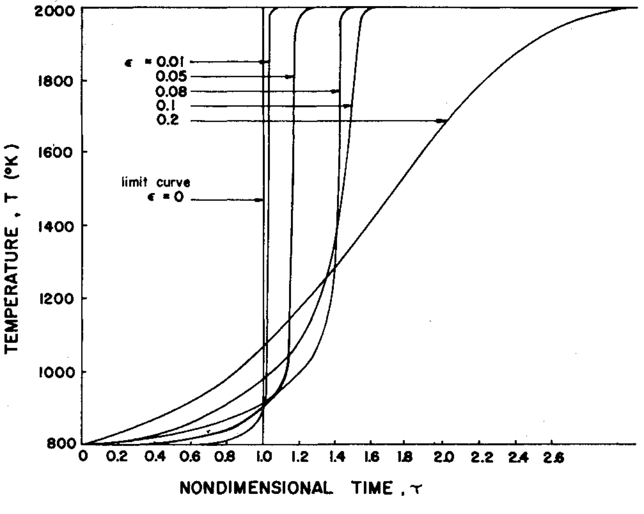

Fig. 3 Effect of parameter $\varepsilon = RT_o/E$ on the temperature variation with time.

$$\hat{t} = \left[Ei\left(\frac{1}{\varepsilon} - \frac{\theta}{1+\varepsilon\theta}\right) - Ei\left(\frac{1}{\varepsilon}\right)\right] - e^{\gamma/\varepsilon(1+\gamma)} \times \left[Ei\left(\frac{1}{\varepsilon(1+\gamma)} - \frac{\theta}{1+\varepsilon\theta}\right) - Ei\left(\frac{1}{\varepsilon(1+\gamma)}\right)\right] \tag{27}$$

Equation (27) is shown in Fig. 1 for $\varepsilon = 0.07944$ for the purpose of comparison with a numerical solution obtained by Zeldovich and Voevodskii.[3] All results correspond to $T_o = 800°F$, $T_f = 2000°F$ and $\gamma = \frac{2}{3}$.

The effect of ε on the exact solution for $n = 1$, Eq. (27), is shown in Fig. 3. As $\varepsilon \to 0$, the solution appears to tend to the limit solution

$$\theta = T_f\, H(\tau - 1) \tag{28}$$

where H is the Heaviside function.

For the case $n = 2$, Eq. (23) can be written as

$$d\tau = -\left(\frac{\alpha}{\gamma}\right)^2 e^{-1/\varepsilon + \alpha}\frac{e^{(\theta_e - \alpha)}}{(\theta_e - \alpha)^2}\,d\theta_e \tag{29}$$

which upon integration and the employment of the initial condition (26) becomes

$$\tau = \frac{1}{\varepsilon^2(1+\gamma)^2}\left\{\exp\left[-\frac{1}{\varepsilon(1+\gamma)}\right]\left[Ei\left(\frac{1}{\varepsilon(1+\gamma)}\right) - Ei\left(\frac{1}{\varepsilon(1+\gamma)} - \frac{\theta}{1+\varepsilon\theta}\right)\right] + \frac{\exp(-\theta/1+\varepsilon\theta)}{[1/\varepsilon(1+\gamma)] - \theta/1+\varepsilon\theta} - \varepsilon(1+\gamma)\right\} \tag{30}$$

Similarly for the case $n = 3$, we obtain

$$\tau = \frac{1}{2\varepsilon^3(1+\gamma)^4}\exp\left[-\frac{1}{\varepsilon(1+\gamma)}\right][\gamma + 2\varepsilon(1+\gamma)] \times \left[Ei\left(\frac{1}{\varepsilon(1+\gamma)}\right) - Ei\left(\frac{1}{\varepsilon(1+\gamma)} - \frac{1}{1+\varepsilon\theta}\right)\right] + \frac{1}{2\varepsilon^3(1+\gamma)^4} \times \left\{\gamma\frac{\exp(-\theta/1+\varepsilon\theta)}{\{[1/\varepsilon(1+\gamma)] - \theta/1+\varepsilon\theta\}^2} - [\varepsilon(1+\gamma)]^2 + [\gamma + 2\varepsilon(1+\gamma)]\frac{\exp(-\theta/1+\varepsilon\theta)}{[1/\varepsilon(1+\gamma)] + (\theta/1+\varepsilon\theta)} - \varepsilon(1+\gamma)\right\} \tag{31}$$

In general, for any integer value of n one could, through subsequent integration by parts, obtain the exact solution of Eq. (3) which would be composed of two exponential integral functions and a finite sum of exponential terms.

Summary and Conclusion

The exact analytical solutions of the exact and approximate differential equations governing the adiabatic explosion of a homogeneous mixture of gases in constant volume vessel of initially uniform temperature [Eqs. (3) and (6), respectively] have been obtained. They are given, for any integer n, in terms of two exponential integral functions and a sum of exponential terms. The exact analytical solution of the exact equation is in complete agreement with the numerical solution obtained by Zeldovich and Voevodskii.[3]

Comparison of the exact solutions with previously obtained approximate solutions, shows that the Zeldovich expansion fails to yield a uniformly valid solution for the long time, high temperature region [$\theta = 0(1/\varepsilon)$] and the PLK method yields solutions which, while accurate for times and temperatures of order unity or less, fail to show the asymptotic behavior of the analytical solution for long times.

Finally, the analytical solution rather nicely reflects the physics of the explosion process for various values of the parameter ε. That is, for small ε (large activation energy or small initial temperature), the analytical solution shows larger delays for the critical conditions to be reached and therefore a longer time is required for the complete consumption of the fuel.

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Influence of Sound upon Separated Flow over Wings

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Introduction

A TECHNIQUE for increasing the lift coefficient and stall angle and decreasing the drag coefficient of a wing at moderate Reynolds numbers by radiating the airstream with sound is described. The sound waves cause premature boundary-layer transition, which reduces separated flow regions and suppresses stall. The technique is expected to be most useful at low Reynolds numbers where the separation of a laminar boundary layer normally would occur.

It has long been known that the turbulence level in a wind tunnel has a strong influence on the transition Reynolds number for the flow about a particular body, such as a sphere. Schubauer and Skramstad, in their classic work on boundary-layer transition on a flat plate,[1] were perhaps the first to recognize that sound of particular frequencies and intensities can also influence the transition process. Most succeeding sound studies have either involved an examination of the influence of sound on the transition process or the development of techniques to prevent sound from causing premature transition in laminar boundary layers on wings, thereby increasing the skin friction drag. Only recently have several investigators realized that sound can be used to control the flow about wings, resulting in an increase of the lift coefficient at high angles of attack. Preliminary measurements of the influence of external sound on wing properties at high angles of attack will be presented in this Note.

Previous Work on Audio Boundary Layer Control

Studies of the influence of sound on boundary-layer transition were performed by Schubauer and Skramstad,[1] Bergh,[2] Boltz, et al.,[3] Brown,[4,5] Jackson and Heckl,[6] Knapp and Roache,[7] and Spangler and Wells.[8] Pfenninger and Reed,[9] on the other hand, were concerned with determining the maximum sound levels which could be allowed and still maintain laminar flow on wings with suction boundary-layer control. Their studies were initiated in the early 1950's when it was noticed that sound propagating through the suction ducts would initiate boundary-layer transition.

These studies have led to a number of conclusions concerning the influence of sound on boundary layer transition. 1) Sound can initiate premature transition, causing the position of transition to move upstream.[1,3] This will happen when flows with or without a pressure gradient are radiated.[3–5] Sound is effective in causing transition up to $Re = 10^7$ (Ref. 9) and $M = 0.5$ (Ref. 3). 2) Only certain sound frequencies can initiate premature transition, the most effective ones being those on Branch II of the neutral stability curve.[1,3] 3) The amount of sound influence depends upon the sound intensity in a manner similar to the influence of the level of turbulence on transition.[1] 4) Both external sound and sound emitted internally through holes in the body surface will initiate transition.[1,9,10]

Recently several investigators have noted benefits which could result from the use of sound to control the boundary layer and, in particular, the stall or separation phenomena. Chang[11] noted that sound could be used to reduce airfoil drag by up to 20%, the power saving due to drag reduction being as much as 19 times greater than the amount of sound power required. The drag reduction had a smooth and continuous dependence upon Strouhal number. This work was performed at a rather low Reynolds number (8×10^4). Brown,[4,5] at Notre Dame, noticed that shear layer vortices could be controlled by sound within rather wide limits. He demonstrated that sound of proper frequency could be used to close the wake behind spheres,

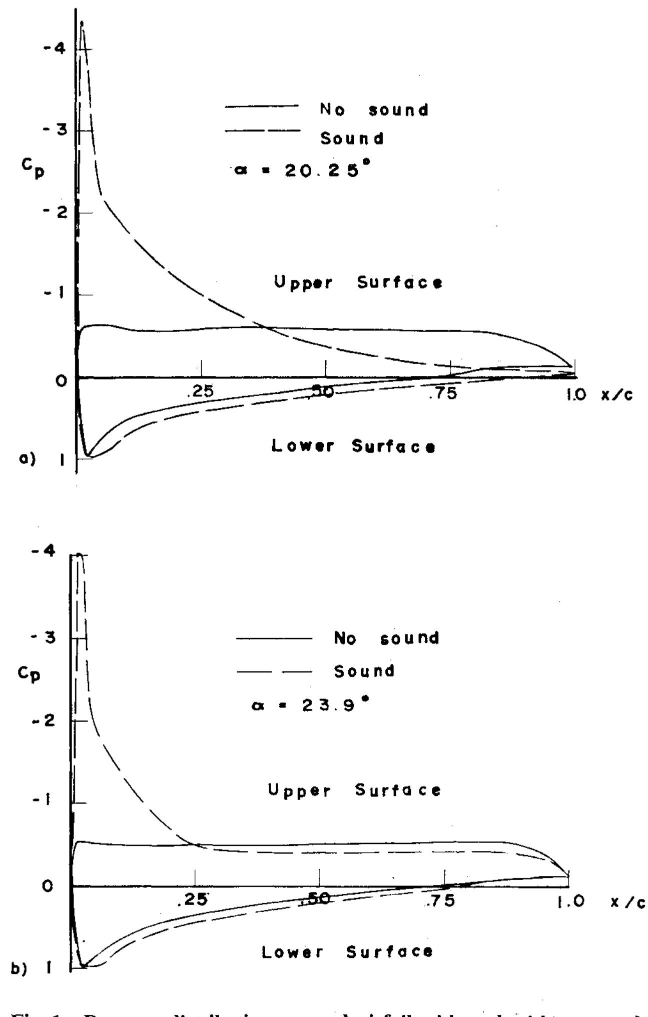

Fig. 1 Pressure distributions around airfoil with and without sound, $Re = 5.3 \times 10^5$. a) No sound properties: $C_L = 0.620$, $C_D = 0.247$, $C_M = -0.089$; sound properties: 7996 Hz, 88 db, $C_L = 0.844$, $C_D = 0.127$, $C_M = 0.042$. b) No sound properties: $C_L = 0.567$, $C_D = 0.283$, $C_M = -0.083$; sound properties: 2401 Hz, 134 db, $C_L = 0.832$, $C_D = 0.233$, $C_M = -0.042$.

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